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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2019 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2019

ගණිතය II
கணிதம் II
**Mathematics II**

**07 E II**

**29.08.2019 / 0830 – 1140**

පැය තුනයි
மூன்று மணித்தியாலம்
***Three hours***

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
**Additional Reading Time - 10 minutes**

Use **additional reading time** to go through the question paper, select the questions and decide on the questions that you give priority in answering.

**Index Number** | | | | | | | |

## Instructions:

- ※ *This question paper consists of two parts;*
  **Part A** (Questions **1 – 10**) and **Part B** (Questions **11 – 17**).
- ※ **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
- ※ **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided.*
- ※ *At the end of the time allotted, tie the answer scripts of the two parts together so that **Part A** is on top of **Part B** and hand them over to the supervisor.*
- ※ *You are permitted to remove **only Part B** of the question paper from the Examination Hall.*
- ※ *Statistical tables will be provided.*

**For Examiners' Use only**

| (07) Mathematics II | | |
|---|---|---|
| Part | Question No. | Marks |
| A | 1 | |
| | 2 | |
| | 3 | |
| | 4 | |
| | 5 | |
| | 6 | |
| | 7 | |
| | 8 | |
| | 9 | |
| | 10 | |
| B | 11 | |
| | 12 | |
| | 13 | |
| | 14 | |
| | 15 | |
| | 16 | |
| | 17 | |
| | Total | |

**Total**

| | |
|---|---|
| In Numbers | |
| In Words | |

**Code Numbers**

| | | |
|---|---|---|
| Marking Examiner | | |
| Checked by: | 1 | |
| | 2 | |
| Supervised by: | | |

[see page two

**Part A**

1. Find all real values of $x$ satisfying the inequality $1 \le \dfrac{3x}{x^2 - 4}$.

2. Shade the region in the $xy$-plane satisfying the inequalities $x^2 + y^2 \le 16$, $x^2 \le 6y$ and $y \le x + 4$.

[see page three

3. Express $2\cos\left(x-\frac{\pi}{3}\right)-\sin x+\sqrt{3}\cos x$ in the form $R\cos(x-\alpha)$, where $R(>0)$ and $\alpha\left(0<\alpha<\frac{\pi}{2}\right)$ are real constants to be determined.

4. Find the values of the real constants $A$ and $B$ such that $\frac{1}{x^2(x+1)}=\frac{A}{x^2}+\frac{B}{x}+\frac{1}{x+1}$ for $x \neq -1, 0.$

**Hence**, evaluate $\int_1^2 \frac{1}{x^2(x+1)}\,dx$ .

[see page four

**5.** Using the method of **integration by parts**, find $\int (2x+1)(\ln x)^2 \, dx$.

**6.** The probability mass function of a discrete random variable $X$ is given below:

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X=x)$ | 0.2 | 0.2 | 0.3 | 0.3 |

Find $E(X)$.

Let $Y$ be the random variable given by $Y = 2X - 3$. Find $E(Y)$ and the probability that $Y$ is positive.

[see page five

7. A box has two red pens, two blue pens and a black pen. Two pens are randomly selected without replacement. Find the probability that both pens selected are of

(i) the same colour,

(ii) different colours.

8. Suppose that $A$ and $B$ are exhaustive events of a sample space $S$. If $P(A) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{5}$, find (i) $P(B)$, (ii) $P\left(A \middle| B\right)$, (iii) $P\left(A' \middle| B'\right)$.

[see page six

**9**. The probability that a randomly selected seed from a packet germinates is 0.7. If five seeds are randomly selected from the packet for planting, find the probability that

(i) at least one of the seeds will germinate,

(ii) exactly three seeds will germinate.

**10**. Let $X$ be a continuous random variable with probability density function $f(x)$ given by

$$f(x) = \begin{cases} k(3x-1), & 1 \le x \le 4, \\ 0, & \text{otherwise,} \end{cases}$$

where $k$ is a positive constant.

Find

(i) the value of $k$,

(ii) the mean of $X$.

* *

[see page seven

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අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2019 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2019

ගණිතය II
கணிதம் II
**Mathematics II**

07 E II

**Part B**

* *Answer **five** questions only.*

**11.** A company owns two machines $A$ and $B$ which have different production capacities for high, medium and low grade nails. This company should produce at least 7, 6 and 13 tons per week of high, medium and low grade nails respectively to meet the demand of the market. It costs the company Rs. 10 000 and Rs. 8 000 per day to operate machines $A$ and $B$ respectively.

The following table gives the capacity of production in tons per day of each machine on each grade of nails.

| **Grade of nails** | **Capacity (tons/day)** | |
|---|---|---|
| | ***A*** | ***B*** |
| High | 2 | 1 |
| Medium | 1 | 1 |
| Low | 2 | 3 |

The company wishes to find out the number of days that each machine be operated per week to minimize the total production cost, while meeting the demand.

(i) Formulate this as a linear programming problem.

(ii) Sketch the feasible region.

(iii) Using the graphical method, find the solution of the problem formulated in part (i) above.

(iv) Due to a technical issue, the machine $B$ has to be operated for a week at most twice as many days as the machine $A$ is operated.

Find the increase of total production cost per week, if the company still wishes to minimize the production cost.

**12.** (*a*) Solve the equation $2(\sin 2x + \sin x - \cos x) = 1$ for $0 \le x \le \pi$.

(*b*) Show that $\tan^{-1}2 + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{2}$.

(*c*) In the usual notation, state the **Sine Rule** for a triangle $ABC$.

Show that if $a + c = 2b$, then $\cos(A - C) = 3 - 4\cos B$.

[See page eight

**13.** (*a*) Find the area enclosed by the circle $x^2+y^2=8$ and the curve $x^2=2y$.

(*b*) The following table gives the values of the function $f(x)=\dfrac{2}{5-2x}$ correct to four decimal places, for values of $x$ between 0 and 1 at intervals of length 0.2.

| $x$ | 0.00 | 0.20 | 0.40 | 0.60 | 0.80 | 1.00 |
|---|---|---|---|---|---|---|
| $f(x)$ | 0.4000 | 0.4347 | 0.4762 | 0.5263 | 0.5882 | 0.6667 |

Using **Trapezoidal Rule**, find an approximate value for $I=\int_0^1 \dfrac{2}{5-2x}\,dx$ correct to three decimal places.

**Hence** find an approximate value for $\ln\left(\dfrac{5}{3}\right)$.

**14.** The mean and the standard deviation of the set of values $\{x_i : i=1,2,\ldots,n\}$ are $\mu$ and $\sigma$ respectively. Find the mean and the standard deviation of the set of values $\{ax_i+b : i=1,2,\ldots,n\}$, where $a$ and $b$ are constants.

The following table summarises the ages (recorded to the nearest year) at the initial diagnosis of high blood sugar of a group of 70 diabetic patients.

| **Age** | **Number of patients** |
|---|---|
| 10 – 20 | 9 |
| 20 – 30 | 12 |
| 30 – 40 | 32 |
| 40 – 50 | 14 |
| 50 – 60 | 3 |

(i) Using a suitable linear transformation or otherwise, calculate the mean and the standard deviation of the given frequency distribution.

(ii) Find the inter-quartile range of the above distribution.

(iii) Two more patients who were both initially diagnosed with high blood sugar at the age of 55 joined the group. Find the inter-quartile range of the frequency distribution of the initial age of diagnosis of high blood sugar of all 72 patients.

**15.** Fruits are packed in three boxes, $A$, $B$ and $C$, so that box $A$ contains only 7 mangoes, box $B$ contains 4 mangoes and 3 pears and box $C$ contains 5 apples and 2 pears. Suppose that a box is selected at random and 2 fruits are randomly picked one after the other without replacement from the selected box.

Assuming that the selection of each box is equally likely, find the probability that

(i) both selected fruits are mangoes,

(ii) at least one of the selected fruits is a mango,

(iii) both fruits selected are mangoes given that one is a mango,

(iv) fruits are of different kinds.

[See page nine

**16.** The time taken (hours) to successfully complete an activity by each student in a group is known to be normally distributed with a mean of 2 hours and a standard deviation of 0.5 hours. Suppose that the students in this group started the activity at 7.00 am on a particular day and their completion times are independent.

(i) Find the probability that a randomly selected student will successfully complete the activity

(a) before 8.30 am,

(b) in between 8.30 am and 9.30 am.

(ii) Given that a randomly selected student had completed the activity before 8.30 am, find the probability that the student had completed the activity before 8.00 am.

(iii) If two students are randomly selected, find the probability that at least one of them had completed the activity before 8.30 am.

**17.** A continuous random variable $X$ has an exponential distribution with probability density function $f(x)$ given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & , \ x > 0 \\ 0 & , \ \text{otherwise} \end{cases}$$

where $\lambda\ (> 0)$ is a parameter.

Find the mean and the variance of $X$.

The lifetime $X$ of an electric equipment is exponentially distributed with a mean of 2 years. Find the cumulative distribution function of $X$ and **hence** find the median of $X$.
(You may take $e^{-0.7} \simeq 0.5$.)

An equipment is randomly selected. Find the probability that

(i) the life time of the equipment will exceed $1\frac{1}{2}$ years,

(ii) the equipment will fail before 2 years, given that the equipment had lasted more than $1\frac{1}{2}$ years.

(You need not simplify the answers.)

* * *