නව/පැරණි නිර්දේශය – புதிய/பழைய பாடத்திட்டம் – New/Old Syllabus

NEW/OLD

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2019 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2019

උසස් ගණිතය I
உயர் கணிதம் I
Higher Mathematics I

11 E I

28.08.2019 / 0830 – 1140

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use **additional reading time** to go through the question paper, select the questions and decide on the questions that you give priority in answering.

Instructions:

Index Number							

- *This question paper consists of two parts;* **Part A** (Questions **1** - **10**) and **Part B** (Questions **11** - **17**).
- **Part A:** *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
- **Part B:** *Answer **five** questions only. Write your answers on the sheets provided.*
- *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
- *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

For Examiners' Use only

(11) Higher Mathematics I		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Factorize: $a^2(b-c)^3 + b^2(c-a)^3 + c^2(a-b)^3$.

2. Let R be the relation defined on the set of integers $\mathbb{Z}$ by xRy if 7 divides $(x^3 - y^3)$. Show that R is an equivalence relation on $\mathbb{Z}$ and write down the equivalence class of 0.

[see page three

Index Number

3. Let $f(x) = \dfrac{x+2}{x-3}$ for $x \neq 3$.

Write down the range of f and find $f^{-1}(x)$. Also, find $f\left(2f^{-1}(0)\right)$.

4. Show that

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

[see page four

5. Show that the line $y = \frac{1}{3}x + 3a$ touches the parabola $y^2 = 4ax$, at a point P such that $AP = 10a$, where A is the focus of the parabola.

Show also that the area of the triangle OAP is $3a^2$, where O is the origin.

6. Let $a, b \in \mathbb{R}$ and let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function given by

$$f(x) = \begin{cases} a\left(1 + e^{-\frac{1}{x}}\right) & \text{if} \quad x > 0, \\ 2 & \text{if} \quad x = 0, \\ \dfrac{\sqrt{1+bx} - 1}{x} & \text{if} \quad x < 0. \end{cases}$$

It is given that $f(x)$ is continuous at $x = 0$. Find the values of a and b.

[see page five

7. Let $f(x) = \begin{cases} x^2+3x+3 & \text{if} \quad x \le 1, \\ 5x+2 & \text{if} \quad x > 1. \end{cases}$

Show that $f(x)$ is differentiable at $x = 1$ and write down $f'(x)$ for $x \in \mathbb{R}$.

Is $f'(x)$ differentiable at $x = 1$?

8. Solve the differential equation $\frac{dy}{dx} - y\cot x + 3\sin^2 2x = 0$, subject to the condition that $y = 1$ when $x = \frac{\pi}{4}$.

[see page six

9. Let f and g be real valued functions defined on the interval $[0, 2]$ such that f' and g are both continuous on $[0, 2]$ and $xf'(x) = g(2 - x)$ for all $x \in [0, 2]$. If $f(2) = 1$ and $\int_0^2 f(x)\,dx = 3$, find $\int_0^2 g(x)\,dx$.

10. Sketch the curves given in polar coordinates by $r = 2\cos\theta$ and $r(\cos\theta + \sin\theta) = 1$ in the same diagram. Find the polar coordinates of their points of intersection.

Department of Examinations Sri Lanka

[see page seven

නව/පැරණි නිර්දේශය – புதிய/பழைய பாடத்திட்டம் – New/Old Syllabus

NEW/OLD

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2019 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2019

උසස් ගණිතය I
உயர் கணிதம் I
Higher Mathematics I

11 E I

Part B

∗ Answer **five** questions only.

11. (*a*) Let A, B and C be subsets of a universal set S. Stating clearly any result in Algebra of Sets that you use, show that

(i) $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$,

(ii) $A \setminus (B \cap C) = (A \setminus B) \cap (A \setminus C)$

Using an example, show that $A \setminus (B \cup C) \neq (A \setminus B) \cup (A \setminus C)$.

(*b*) In a competition, a school awarded medals for three different categories as follows:

45 medals were awarded for dancing,

21 medals were awarded for singing, and

27 medals were awarded for sports.

If these medals were awarded to a total of 54 recipients and only 13 persons received medals in all three categories, how many persons received medals, exactly in two of these categories?

12. (*a*) Let $a, b, c \in \mathbb{R}^+$.

Using Arithmetic Mean – Geometric Mean inequality, show that

$$\frac{a}{b} + \frac{b}{a} \geq 2.$$

Hence, show that

(i) $\dfrac{b+c}{a} + \dfrac{c+a}{b} + \dfrac{a+b}{c} \geq 6$,

(ii) $x^2\left(1+y^2\right) + y^2\left(1+z^2\right) + z^2\left(1+x^2\right) \geq 6xyz$.

(*b*) The transformation $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$, maps points in the xy-plane into points in the $x'y'$-plane. Find the value of m in order that the line $y = mx + c$, $\left(m \neq \frac{2}{3}\right.$ and $\left.c \neq 0\right)$, is invariant under the above transformation.

Let $A \equiv (c, 0)$ and $B \equiv (0, c)$ be two points in the xy-plane. Find the coordinates of their images A' and B' under this transformation and verify that the points A' and B' lie on the line $x' + y' = c$.

[see page eight

13. State and prove **De Moivre's Theorem** for a positive integral index.

Using **De Moivre's Theorem**, show that

(i) $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$, and

(ii) $\sin 5\theta = \sin^5\theta - 10\cos^2\theta\sin^3\theta + 5\cos^4\theta\sin\theta$.

Deduce that $\tan 5\theta = \dfrac{\tan\theta\left(\tan^4\theta - 10\tan^2\theta + 5\right)}{\left(1 - 10\tan^2\theta + 5\tan^4\theta\right)}$.

Solve the equation $\tan 5\theta = 0$ for $0 < \theta < \dfrac{\pi}{2}$, and show that $\tan^2\left(\dfrac{\pi}{5}\right)$ and $\tan^2\left(\dfrac{2\pi}{5}\right)$ are the roots of the equation $x^2 - 10x + 5 = 0$.

Hence, show that $\sec^2\left(\dfrac{\pi}{5}\right) + \sec^2\left(\dfrac{2\pi}{5}\right) = 12$.

14.(*a*) Let C_1 and C_2 be the curves given by $y = \dfrac{4x}{1+x}$ and $y = \dfrac{2}{3}x^2$ for $x \in \mathbb{R}$, respectively. Find the coordinates of the points of intersection of C_1 and C_2.

Sketch the graphs of C_1 and C_2 in the same figure showing clearly the asymptotes and turning points (if any). Find the area enclosed by C_1 and C_2.

Find also the volume of the solid generated by revolving the area enclosed by the curves C_1 and C_2 through 4-right angles about the x-axis.

(*b*) Solve the differential equation:

$$2x^2\frac{dy}{dx} - 2xy + y^2 = 0.$$

15.(*a*) Let $I_n = \displaystyle\int_0^{2\pi} \sin^n(x+\alpha)\,dx$, where α is a real constant and n is an integer such that $n \geq 2$.

Show that, $nI_n = (n-1)\,I_{n-2}$ for $n \geq 2$.

Hence, find the value of $\displaystyle\int_0^{2\pi}\left(\sqrt{3}\sin x + \cos x\right)^6 dx$.

(*b*) Let $y = \tan(e^{2x} - 1)$.

Show that $\dfrac{d^2y}{dx^2} = 2\dfrac{dy}{dx}\left(1 + ye^{2x}\right)$.

Hence, find the Maclaurin series expansion of y up to and including the term involving x^4.

16. Show that the equation of the tangent to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ at the point (x_1, y_1) is $\dfrac{xx_1}{a^2} + \dfrac{yy_1}{b^2} = 1$.

Deduce the equation of the tangent to the ellipse at the point P with eccentric angle θ, and show that the normal to the ellipse at P is given by $(a\sec\theta)x - (b\,\mathrm{cosec}\,\theta)y = a^2 - b^2$.

Let T and T' be the points where the tangent meets the OX and OY axes, respectively, and let N and N' be the points where the normal meets the OX and OY axes, respectively.

(i) Show that the equation of the locus of the mid-point of NN' as θ varies is $4(a^2x^2 + b^2y^2) = (a^2 - b^2)^2$.

(ii) Find the value of the eccentric angle θ $\left(0 < \theta < \dfrac{\pi}{2}\right)$ at which the lines TT' and NN' are equally inclined to the two axes of coordinates. In this case, find $(TT')(NN')$ in terms of a and b.

[see page nine

17. (*a*) Let $f(x) = \dfrac{\sin 2x}{2+\cos 2x}$ for $x \in \mathbb{R}$.

(i) Show that $-\dfrac{1}{\sqrt{3}} \leq f(x) \leq \dfrac{1}{\sqrt{3}}$ for $x \in \mathbb{R}$.

(ii) Sketch the graph of $y = f(x)$ for $0 \leq x \leq \pi$.

(*b*) **Using Simpson's Rule** with the values of e^{-x^2} given in the following table, find an approximate value for $\int_0^1 e^{-x^2} \mathrm{d}x$:

x	0	0.25	0.50	0.75	1.0
e^{-x^2}	1	0.9394	0.7788	0.5698	0.3679

Deduce an approximate value for $\int_0^1 e^{\left(\ln 2 - 9x^2\right)} \mathrm{d}x$.

***

නව/පැරණි නිර්දේශය – புதிய/பழைய பாடத்திட்டம் – New/Old Syllabus

NEW/OLD

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2019 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2019

උසස් ගණිතය II
உயர் கணிதம் II
Higher Mathematics II

11 E II

31.08.2019 / 1300 - 1610

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use **additional reading time** to go through the question paper, select the questions and decide on the questions that you give priority in answering.

Instructions:

Index Number							

* *This question paper consists of two parts;* **Part A** (Questions **1 - 10**) and **Part B** (Questions **11 - 17**).
* **Part A:** *Answer* ***all*** *questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:** *Answer* ***five*** *questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove* ***only*** **Part B** *of the question paper from the Examination Hall.*

* *Statistical Tables will be provided.*
* *g denotes the acceleration due to gravity.*

For Examiners' Use only

(11) Higher Mathematics II		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

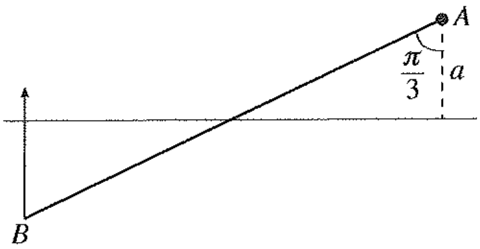
1. The position vectors of three points A, B and C with respect to a fixed origin O are $\mathbf{i}+2\mathbf{j}-\mathbf{k}$, $2\mathbf{i}-\mathbf{j}+3\mathbf{k}$ and $7\mathbf{i}+\alpha\mathbf{j}+\beta\mathbf{k}$, respectively. Find the values of the constants α and β such that the vector $\overrightarrow{OC}$ is perpendicular to the plane OAB.

2. A force $\mathbf{F}$ of magnitude 6 N acts along the line whose vector equation is $\mathbf{r}=\mathbf{i}+2\mathbf{j}-4\mathbf{k}+\lambda(\mathbf{i}+2\mathbf{j}-2\mathbf{k})$, where λ is a scalar parameter. If the distances are measured in metres, show that the vector moment $\mathbf{M}$ of $\mathbf{F}$ about the origin is of magnitude $4\sqrt{5}\,\mathrm{N\,m}$ and that $\mathbf{M}\cdot\mathbf{k}=0$.

[see page three

0148

3. A uniform rod AB of length $4a$ and density ρ is smoothly hinged at the end A to a fixed point at a height a above the free surface of a homogeneous liquid of density $\sigma\left(<\frac{4\rho}{3}\right)$. The rod is kept in equilibrium, making an angle $\frac{\pi}{3}$ with the downward vertical as shown in the figure, by a vertical light inextensible string attached to the end B. Find the tension in the string.

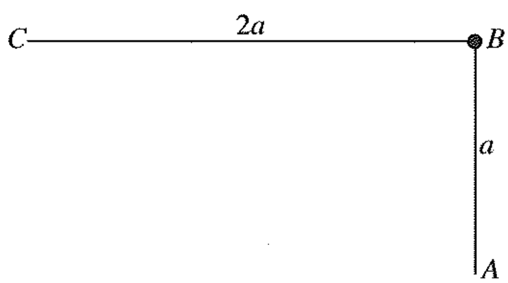


4. With respect to a fixed origin, the position vector $\mathbf{r}$ of a particle P at time t is given by $\mathbf{r} = a(\omega t - \sin \omega t)\,\mathbf{i} + a(\omega t - \cos \omega t)\,\mathbf{j}$, where a and ω are positive constants and $0 \leq \omega t \leq \pi$. Find the velocity vector $\mathbf{v}$ and the acceleration vector $\mathbf{f}$ of P, at time t. Find the time at which $\mathbf{v} \cdot \mathbf{f} = 0$, and show that the speed of P at this instant is $a\omega\left(\sqrt{2}-1\right)$.

[see page four

5. Two small smooth spheres P and Q of masses m and $2m$ respectively, are moving on a smooth horizontal table and collide with each other. The velocities of P and Q, just before impact are $2\mathbf{i} + \mathbf{j}$ and $-2\mathbf{i} + 3\mathbf{j}$ respectively. The coefficient of restitution between P and Q is $\frac{1}{3}$. Write down equations sufficient to determine the velocities of P and Q just after impact.

6. A frame is formed by rigidly attaching a uniform rod AB of mass m and length a to a uniform rod BC of mass $2m$ and length $2a$, in such a way that angle $A\hat{B}C$ is a right angle. The frame can rotate freely about a fixed smooth horizontal axis through B and perpendicular to the plane of the frame. The frame is released from rest in the position in which BC is horizontal and A is below B. Assuming that the moment of inertia of the frame ABC about the axis of rotation is $3ma^2$, find the angular velocity of the frame when the rod BC is vertical with C below B.



[see page five

7. A box contains 10 red and 15 green balls which are identical in all aspects other than colour. Balls are drawn from this box at random, one by one, with replacement.

(i) Calculate the probability that the first red ball is drawn on or before the 3rd draw.

(ii) Calculate the conditional probability of drawing first green ball on the 8th draw, given that first 5 balls drawn are red.

8. The number of misprints per page of a certain document follows a Poisson distribution with mean 2.1. Find the probability that a randomly selected page has

(i) exactly 1 misprint,

(ii) at least 3 misprints.

[see page six

9. Let $f(x) = \begin{cases} kx\left(a - x^2\right), & 0 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases}$

Find the values of constants k and a in order that $f(x)$ is the probability density function of a continuous random variable X whose mean is $\frac{8}{15}$. Show that the standard deviation of X is $\frac{\sqrt{11}}{15}$.

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

10. The cumulative distribution function $F(x)$ of a discrete random variable X is given by $F(x) = \frac{1}{16}\left(8x - x^2\right)$ for $x = 1, 2, 3, 4$. Obtain the probability mass function of X and find $E(X)$.

....

....

....

....

....

....

....

....

....

....

....

....

....

....

[see page seven

Department of Examinations Sri Lanka

නව/පැරණි නිර්දේශය – புதிய/பழைய பாடத்திட்டம் – New/Old Syllabus

NEW/OLD

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2019 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2019

උසස් ගණිතය II
உயர் கணிதம் II
Higher Mathematics II

11 E II

Part B

※ Answer **five** questions only.

11. A system consists of three forces acting at points having position vectors, with respect to an origin O, as given in the following table.

Point	Position vector	Force
A_1	$\mathbf{r}_1 = 2\mathbf{i} - 4\mathbf{j}$	$\mathbf{F}_1 = \mathbf{i} + 4\mathbf{j} - \mathbf{k}$
A_2	$\mathbf{r}_2 = \mathbf{j} - 3\mathbf{k}$	$\mathbf{F}_2 = -3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$
A_3	$\mathbf{r}_3 = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$	$\mathbf{F}_3 = -\mathbf{i} - \mathbf{j} + 2\mathbf{k}$

By introducing forces $\pm\mathbf{F}_s$, $s = 1, 2, 3$, at the origin O, show that the given system can be reduced to a single force $\mathbf{R} = \sum_{s=1}^{3} \mathbf{F}_s$ acting at O together with a couple of vector moment $\mathbf{G} = \sum_{s=1}^{3} \mathbf{r}_s \times \mathbf{F}_s$.
Find vectors $\mathbf{R}$ and $\mathbf{G}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$.

Deduce that the system is equivalent to a single resultant force $\mathbf{R}$ of magnitude $\sqrt{26}$.

Show that the lines of action of $\mathbf{F}_1$ and $\mathbf{F}_2$ meet at a certain point A_0 with position vector $\mathbf{r}_0$, where $\mathbf{r}_0$ is to be determined. Verify that the line of action of $\mathbf{F}_3$ also passes through the point A_0.
Write down the equation of the line of action of the single resultant force $\mathbf{R}$ in the form $\mathbf{r} = \mathbf{r}_0 + \gamma\mathbf{R}$.
Find the position vector of the point where this line meets the xy-plane.
Hence, show that the Cartesian equations of the line of action of the resultant force $\mathbf{R}$ may be obtained as $\frac{x-6}{3} = \frac{y+4}{-4} = \frac{z}{1}$.
Show further that the Cartesian equation of the plane on which the given system of forces lies is obtained as $x + 3z = 0$.

12. A lamina in the shape of a trapezium $ABCD$ with AB parallel to DC, $AB = 3a$, $DC = a$ and $B\hat{A}D = A\hat{B}C = \frac{\pi}{4}$ is immersed vertically in a homogeneous liquid such that AB is on the free surface of the liquid.
Show that the centre of pressure of the lamina $ABCD$ is at a distance $\frac{3a}{5}$ vertically below the mid-point E of AB.
A door in the shape of the above lamina $ABCD$ is made on a vertical side of a tank with AB horizontal and CD below AB. The door is smoothly hinged along CD. The tank is filled to the level of AB with a homogeneous liquid of density ρ.
Find the least force that should be applied at E in order to keep the door closed with liquid inside the tank.

[see page eight

13. An engine pulls a train along a straight horizontal track against a resistance which at any time is k times the momentum of the train, where k is a constant. The engine works at constant power $9Mkv_0^2$, where M is the total mass of the engine and the train. Show that

(i) the maximum speed that the train can attain is $3v_0$.

(ii) the time taken for the train to increase the speed from v_0 to $2v_0$ is $\frac{1}{2k}\ln\left(\frac{8}{5}\right)$.

When the train is moving with speed U its power is cut off and a constant braking force of magnitude F is applied, in addition to the above resistance. Show that the train will stop at a time $\frac{1}{k}\ln\left(\frac{F+MkU}{F}\right)$ after the power is cut off.

14. A particle P of mass m resting on a smooth horizontal table is connected to a fixed point O on the table by a light elastic string of natural length a and modulus of elasticity mg. When time $t = 0$, the particle P is at a distance a from O, with the string just taut, and P is projected along the table in a direction perpendicular to the initial line of the string, with velocity of magnitude $U = 2\sqrt{\frac{ga}{3}}$.

Using the principle of conservation of energy and the principle of conservation of angular momentum about O, show that

$$\left(\frac{dr}{dt}\right)^2 = U^2\left(1-\frac{a^2}{r^2}\right)-\frac{g}{a}(r-a)^2.$$

Deduce that

(i) the maximum length of the string is $2a$, and the tension in the string at this instant is mg,

(ii) the speed of the particle at this instant is $\frac{U}{2}$.

Find $\frac{d^2r}{dt^2}$, in terms of r and a when $\frac{dr}{dt} \neq 0$.

15. Show that

(i) the moment of inertia of a uniform hollow circular cylinder, of mass M and radius a about its axis is Ma^2, and

(ii) the moment of inertia of a uniform circular disc of mass m and radius a about the axis through its centre perpendicular to its plane is $\frac{1}{2}ma^2$.

A closed container C made from a thin uniform metal sheet, consists of a right hollow circular cylinder of radius a and length $3a$ to which two uniform circular discs, each of radius a, are attached at the two ends. Show that the radius of gyration, k of the container C about its axis is given by $k^2 = \frac{7}{8}a^2$.

The container rolls, without slipping, down a rough plane of inclination α to the horizontal, with its axis horizontal and perpendicular to the lines of greatest slope of the plane.

Show that the acceleration f of the container C, in this motion, is given by $f = \frac{8}{15}g\sin\alpha$, and that the coefficient of friction μ between the container and the plane is such that $\mu > \frac{8}{15}\tan\alpha$.

[see page nine

16. (*a*) Let X be the number of vehicles leaving a certain car park during a five minutes interval. Suppose X has the following probability distribution:

x	1	2	3	4	5	6
$P(X=x)$	p	$2p$	$3p$	$3p$	$2p$	p

Find the value of p and the expected value $E(X)$ of X.

Show that the standard deviation of X is $\frac{\sqrt{7}}{2}$.

The random variable Y is defined by $Y = 2X + 3$. Find the expected value $E(Y)$ of Y and the standard deviation of Y.

Also, find the value of $P(Y \geq E(Y))$.

(*b*) The probability that a patient recovers from a delicate surgery is $\frac{2}{5}$. Five patients were monitored at random, after they underwent this surgery.

Find the probability that

(i) at least 3 recover,

(ii) exactly 2 recover,

(iii) none recovers.

17. (*a*) The lifetime, T hours, of a certain kind of electric lamp can be modelled by the probability density function

$$f(t) = \begin{cases} \frac{1}{a} e^{-\left(\frac{1}{b}\right)t}, & t \geq 0 \\ 0 \quad , & \text{otherwise} \end{cases}$$

where a and b are positive constants.

Show that $a = b$.

It is given that 40% of lamps have a lifetime longer than 2000 hours. Find the common value of a and b.

Find the distribution function of T, and **hence**, show that $P(T > t + c \mid T > c) = P(T > t)$, where $t \geq 0$ and c is a positive constant.

(*b*) The speeds of vehicles passing a certain point A on a motorway can be taken to be normally distributed. Observations show that of vehicles passing the point A, 95% are travelling at speed less than 85 km h^{-1} and 10% are travelling at less than 55 km h^{-1}.

(i) Find the average speed of the vehicles passing the point A.

(ii) Find the percentage of vehicles that travel at a speed more than 70 km h^{-1}.

***